Are Patents Discouraging Innovation?

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Abstract

The strengthening of the U.S. patent regime in the early eighties was followed by a sharp increase in patenting and a rather small increase in the in R&D expenditure in some industries in the U.S. This "patent paradox" is prominently observed in complex industries, like the semiconductor industry. In this paper I develop a model of invention and product development to examine the effects of a change in the patent regime on the patenting and R&D decisions of firms in complex industries. Firms in these industries have a greater need to access a large number of ideas to successfully develop a final product. I consider two different environment — one without licensing and one with licensing. While a stronger patent regime leads to higher patenting and higher R&D investments in both environments, the strategic complementarity between patenting and R&D is relatively weaker in the presence of licensing. A change in the patent regime that strengthens intellectual property (IP) laws creates incentives for firms to increase patenting. Whether that will lead to a similar increase in R&D investment will depend on the licensing environment. (*JEL* L00, L24, O34)

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1 Introduction

In 1982 the U.S. Congress established the Court of Appeals for the Federal Circuit (CAFC), a move seen as strengthening patent protection in the United States. The intention on the part of the Government was to create a strong intellectual property regime that will create incentives for firms to conduct R&D. Researchers have debated the pros and cons of this change.[2] Hall and Ziedonis (2001) have observed that in the semiconductor industry patenting activity has increased substantially after 1982 while R&D expenditure has maintained the previous trend. They and other researchers have suggested that the strengthening of the patent regime has not changed the incentive to perform R&D significantly, but has provided incentives to create large patent portfolios for bargaining purposes. This observation is particularly relevant for "complex product" industries, such as the semiconductor industry, where the development of the end product is generally achieved by using ideas and products owned by different firms.[3]

Some complex product industries, like electronics and semiconductors, have traditionally relied on licensing and bilateral bargaining (called cross-licensing) to access the knowledge owned by different firms. The number of patented (or potentially patentable) ideas or inventions needed to develop the products produced by these industries is large and various key patents are often owned by different firms. Grindley and Teece (1997) note that "with this degree of over-

[2]See Cohen, Nelson and Walsh (2000), Kortum and Lerner (1998), Merges and Nelson (1990).

[3]The term "complex product" is used by Cohen, Nelson and Walsh (2000) to describe "commercializable product or process ... comprised of numerous separately patentable elements.... In complex product industries, firms often do not have propriety control over all the essential complementary components of at least some of the technologies they are developing."

lap of technology, companies protect themselves against mutual infringement by cross-licensing portfolios of all current and future patents in a field-of-use, without making specific reference to individual patents.” Researchers, who have surveyed the intellectual property (IP) managers of some of the computer, electronics and semiconductor firms, report that after the strengthening of the patenting regime licensing activities in the industry have increased and firms' patent portfolios have become important bargaining chips in these agreements.

As mentioned before, the reason for strengthening the U.S. patent regime was to create an incentive system that rewards R&D and, thereby, creates incentive for firms to invest more in R&D activities. The "patent paradox", a term used by Hall and Ziedonis (2001), refers to the previously mentioned empirical observation that the change in the U.S. patent regime was followed by substantial increase in patenting activity and only a small increase in R&D activity. The patent paradox raises important questions regarding the role played by the institution of patenting in firms' decision making process.

No one so far, to my knowledge, has attempted to theoretically analyze the link between patenting, inventions and bilateral licensing in the context of a patent regime change. My paper is an attempt in that direction.

This paper presents a stylized model of basic invention, patenting and product development to study the effects of a stronger patent regime on patenting and R&D in a given licensing environment. I treat the process of acquiring basic inventions (or *ideas*) and the process of developing a new product as two separate activities. R&D expenditure determines the number of new in-house ideas (in expected terms) acquired by each firm, but the in-house ideas alone do not guarantee successful product development.[4] Access to additional ideas, that are developed by rival firms, increase the probability of successful product development for each firm. Another feature of the model is that the industry is characterized by a complexity parameter. A complex product industry is one where relatively more ideas are needed to successfully develop the final product. In other words, for a firm in a complex industry, who has access to only a few ideas, the probability that this firm will successfully develop a final product is small. I compare the effects of a stronger patent regime on firms' decision to patent and invest in R&D without and with licensing. The model predicts that in complex industries the responsiveness of a firm's R&D decision to an increase in the strength of the patent regime depends on the licensing environment. I consider two different environments — one with no-licensing and the other with licensing. In the presence of licensing the strategic complementarity between R&D and patenting is weaker. Therefore in response to a strengthening of the patent regime, even when the increase in patenting predicted by both the licensing environments are the same, the R&D expenditure will be less affected in the presence of licensing as compared to the case without licensing.

The result can be understood by observing that in this paper a stronger patent regime encourage patenting by lowering the cost of patenting. Increased patenting activity decreases imitation and increases the returns to firms' own R&D investment in the margin. This provides incentives for higher R&D investment because with increased patenting firms now know that imitation activity will decrease which will allow the owners of innovations a better chance to be the only developer of the final product. This creates the strategic complementarity between

[4]The two words "inventions" and "ideas" are used interchangeably in this paper.

firms' decision to patent and their decision to invest in R&D. The complementarity result hold in both kind of licensing environments considered in this paper. In the absence of licensing, firms can improve their chance of developing the final product either via in-house inventions or via imitation. In the presence of licensing, firms have access to an additional way of obtaining others' inventions — via licensing. In the presence of the licensing each firm still engage in patenting to deter imitation but knows that the rival firm can still access its inventions via licensing. This reduces the complementarity between patenting decision and R&D investment decision made by each firm. A stronger patent regime still provides incentive to engage in higher levels of patenting but the resulting increase in R&D investment is smaller in the presence of licensing. Empirically, therefore, it is possible to observe a stronger patent regime leading to a substantial increase in patenting activity without observing a similar increase in R&D activity.

1.1 Related Literature

There has been substantial theoretical work on cumulative innovation. The innovation process modeled in this paper is not cumulative in the usual sense of the term. In the literature cumulative innovations generally refers to unidirectional complementarity, often time-lagged. The basic idea is that today's inventions are not only valuable for the immediate benefits they provide, but also are valuable inputs for future inventions. For cumulative inventions complementarity exists between today's inventions and tomorrow's inventions. In this paper each firm's access to the industry-wide aggregate inventions is crucial for the development of any new product. The more access a firm has to the aggregate pool of ideas the higher are the chances that it would be able to develop the final product successfully.

Bessen and Maskin (2000) address the issue of complementary innovations and their paper relates most closely to my work. They investigate a firm's incentive to invest in R&D by comparing two scenarios, one without a patent system and another with a patent system. Complementarity is modeled by assuming that the expected number of inventions increase when both firms invest in R&D. In the no-patent case two firms make the decision whether to invest or imitate. In the presence of a patent system there is no imitation. Instead firms can invest in new innovation only if the patent-holders agree to license their innovations. Bessen and Maskin identify conditions under which patent-holders will not license their innovations and will, therefore, lead to an outcome that reduces social welfare.

This paper develops a model where imitation of inventions, as in Bessen and Maskin (2000), has a positive social value. In Bessen and Maskin, imitation increases the expected number of future innovations. In my paper imitation increases the chance of successfully developing a final product which is valuable to the consumers. The patent system plays an important role by altering the cost of patenting and, thus, altering firm's choice between in-house R&D and outside R&D (obtained either by imitation or licensing).

The three-stage structure in my model is similar to that in Katz and Shapiro (1985). Katz and Shapiro, however, focus on process innovations and do not consider cumulative innovations. They find that firms will license small innovations and effect of licensing on research incentives is ambiguous. In this paper the inventions are in the form of ideas and the two types of licensing environments are taken as given. I obtain the result that the effect of a strong patent regime on R&D in the presence licensing is less pronounced than that in the absence of licensing.

This paper is organized in the following way. In the next section I describe some empirical trends from the U.S. semiconductor industry, some of which this model attempts to address. The baseline model is presented next. In section 4 I discuss the no-licensing equilibrium. In section 5 the planner's problem is described while in section 6 the licensing equilibrium is discussed. Section 7 concludes.

2 Data from the Semiconductor Industry (1979-2002)

Since this paper is about firms in complex-product industry, I look at some of the aggregate data from the semiconductor industry. As mentioned before, the semiconductor industry fits very closely to the notion of a complex-product industry. The development of any final product by a firm in this industry needs access to a lot of potentially patentable inventions, many of which are often patented by different rival firms. Firms in the semiconductor industry has a long history of accessing each others' ideas via imitation and cross licensing (Levin, 1982).

Grindley and Teece (1997) have noted that the strengthening of the U.S. patent regime after 1982 had an impact on firms' patenting strategies in this industry. They observe that "Coincident with the increased importance of patents is the increased importance of licensing and cross-licensing. Cross-licensing has become a significant dimension of competition." The importance of patents have magnified because of their strategic importance in licensing. As a result, semiconductor firms are now patenting more aggressively.

I look at the time series R&D and patenting data for this industry from 1979-2002. The firm level R&D data are obtained from Compustat. After accounting

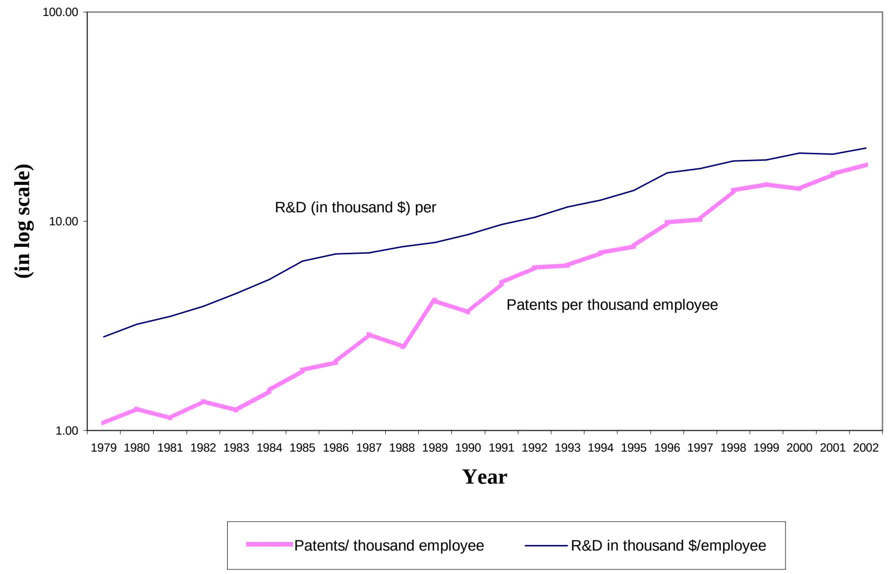


Figure 1: Patents per employee and R&D per employee

for name changes, mergers and exit sample of 164 publicly traded semiconductor firms are considered.

The patent data considered here is the patent application data. This data is obtained from two sources. The patent data for the sample of 164 firms between 1979-1999 are extracted from data file provided by the Hall, et al. Next, new data is collected for these firms from the USPTO website and the two are compiled together to give the patent data set from 1979-2002.

To get an ideas about the growth of patenting and R&D relative to the size of the industry, I look at the patenting per employee and R&D per employee time series. The strengthening of the U.S. patent regime occurred in the mid-1980s. A look at these two time-series data gives us some ideas regarding the aggregate behaviors of this industry before and after the change in the patent regime. The series are shown in figure 1. The size-adjusted data shows that both patenting per

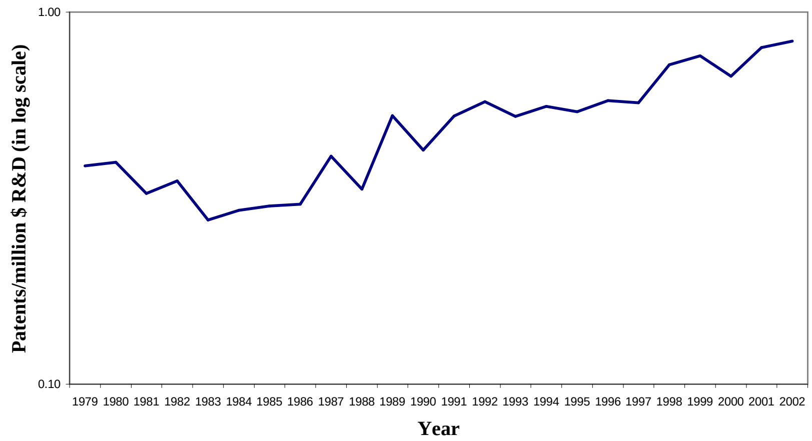


Figure 2: Patent applications per million $ R&D

employee and R&D per employee have grown over time, but the growth rate of patenting per employee is higher than that of R&D per employee. Also the growth rate for patenting per employee have increased after 1982, while that for R&D per employee has slowed down after 1985.

Another way to look at the patenting phenomenon is to study the changes in patenting efficiency, i.e., what is the relative size of patenting relative to R&D. Figure 2 shows the number patent(s) per real R&D $ data for the industry. The data shows that after 1983 (and more prominently after 1986) the total patenting for the industry has risen faster than the total R&D, so that the ratio of the two has changed. This is one more way of showing that after 1983 this industry as a whole is patenting more compared to its aggregate R&D.

The observations made above raise the question whether firms in the semiconductor industry consider patents as means to protect valuable inventions, or do patenting strategies correlate more strongly to other firm-level strategies, like licensing strategy. If patents are only meant to protect inventions then a strong patent regime would create incentives for higher investment in R&D and, hence, we will observe high levels of R&D followed by high levels of patenting. On the other hand if a strong patent regime leads to a high level of patenting without a similar increase in R&D investment that raises the possibility that firms patenting decisions are weakly related to their R&D strategy and, hence, the increase in patenting might be related to completely different firm-level strategies.

This paper presents a model which develops a framework that attempts to provide some understanding of the some of the above mentioned data trends.

3 Basic Model

I focus on an industry with two firms. This is one period model with multiple stages. The consumers care only about the new product and are represented by a demand function, $P = D(Q)$, where Q is the total quantity of the new product demanded at price P. $D(.)$ is a downward sloping, well-behaved demand function, with $D^{-1}(0) < \infty$.

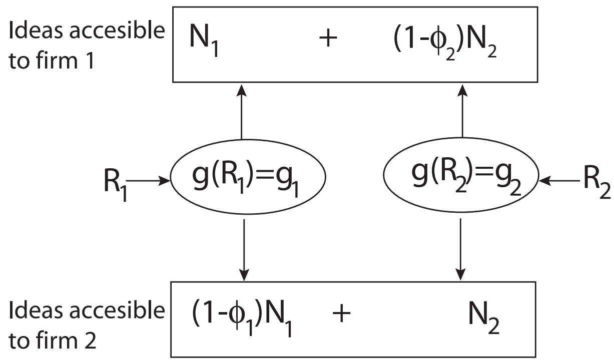
In this model, therefore, firms make a profit if they have a new product at the end of the period. Ideas are developed by firms at an earlier stage and are the building blocks of the new product. The probability of successful product development by each firm depends on the total number of new ideas a firm has access to.

The number of ideas that a firm gets is a random variable. Each firm can acquire ideas in two different ways. It can either invest in R&D and generate in-house ideas or can imitate ideas that have been generated by other firms.

To generate in-house ideas each firm i chooses R&D expenditure, R_i, which affects the distribution of the number of new ideas, N_i, developed by firm i. I assume that $N_i \sim$ Poisson(g_i), where $g_i \equiv g(R_i)$ and $g(.)$ is a monotonically increasing function of R_i. I also assume that N_1 and N_2 are independent random variables and no two ideas are alike.

Firms can also acquire ideas via imitation. Imitation allows a firm to access the other firm's knowledge pool without making a payment. The extent of successful imitation by a firm is determined by the patent strategy adopted by its rival firm. Firms patent to protect their ideas. By protecting ideas, firms make it harder for other firms to imitate. Therefore in this paper patenting is a firm-level decision that reduces imitation. Specifically, patenting is modeled in the following way : firm 1 chooses a patenting strategy, ϕ_1, such that $0 \leq \phi_1 \leq 1$. ϕ_1 is a measure of the strength of firm 1's IP policy. It can be a function of the total number of patents firm 1 receives and its emphasis on hiring lawyers and IP managers to successfully defend its intellectual property. Similarly firm 2 chooses ϕ_2.

Firm 1's patenting choice, ϕ_1, determines how many ideas firm 2 can successfully imitate. I assume that when firm 1 chooses ϕ_1 firm 2 can successfully imitate $(1 - \phi_1)N_1$ of firm 1's ideas. If firm 1 chooses $\phi_1 = 0$ then firm 2 will be able to imitate all of firm 1's ideas. If firm 1 chooses $\phi_1 = 1$, firm 2 will not be able to imitate any ideas. If $0 < \phi_1 < 1$ then firm 2 will be able to imitate some ideas of firm 1's idea but not all. Similarly firm 2's patenting strategy, ϕ_2, determines the number of ideas that can be imitated by firm 1.



Since ϕ can take any value between 0 and 1, this model does not have the restriction that the number of ideas imitated by a firm must be an integer. The assumption of perfect divisibility of ideas simplifies the analysis considerably. One way to interpret this is that ideas are complex entities themselves. An idea might have several different components that work together to generate a particular form of usable knowledge. Usable knowledge of a different form (or quality) may also be obtained by combining some, but not all, of the above mentioned components. Therefore an idea, when transmitted via imitation, may not represent the same unit of knowledge that is available to the original innovator. This model assumes that the IP policy of the firm determines how much of the knowledge content will be transmitted to the imitator.

Firms's R&D and patenting strategies are also affected by the external patent policy environment. The patent regime is parameterized by a non-negative parameter, S. This parameter is taken as given in this model. The patent regime directly affects the cost of patenting. A higher value of S suggests a stronger patent regime, in the sense that it lowers the cost of patenting and, thereby, reduces the cost of enforcing IP rights. A lower value of S will have the opposite effect.

The cost of firm 1's patenting strategy is denoted by a monotonically increasing, convex function, $C(.)$, of ϕ_1. $S > 0$ is a parameter of the cost function, with $\frac{\partial C}{\partial S} < 0$. Also $C(0) = 0$ and $C'(0) > 0$. Firm 2 has an identical cost function for patenting.

Firms are ex-ante symmetric. Each firm chooses R&D and patenting strategies, engages in licensing (only when licensing is allowed) and introduces a new finished product with some probability. There are three stages to the process.

Stage 1 (R&D and IP Protection Stage): Firms choose R&D strategies, R_1 and R_2, and patenting strategies, ϕ_1 and ϕ_2, by maximizing expected profit.

For firm 1, the choice of R_1 affects the expected number of in-house new ideas, N_1, obtained. The choice of ϕ_1 determines firm 1's extent of patent protection. The cost of choosing ϕ_1 is $C(\phi_1)$. Firm 2 faces a similar problem.

At the end of this stage in-house ideas N_1, N_2 and imitated ideas $(1-\phi_2)N_2, (1-\phi_1)N_1$ are realized.

Stage 2 (Licensing Stage): Firms go into the licensing stage knowing $N_1,$ and N_2. To abstract from the discussion of optimal licensing mechanism choice, I assume a licensing structure where each firm is either a licensor (one who gives the license) or licensee (one to whom a license is given) of ideas. A licensor firm permits a licensee firm to use the ideas developed by the former, but licensing does not preclude the licensor firm from using its own ideas. Licensing, therefore, allows firms to access the same ideas simultaneously. This is different from the generally accepted notion of buying and selling goods, where the buyer of the good can exclude the seller from using the good. In this paper, however, I am going to use the words buyer and seller to indicate the licensee and the licenser respectively.

Two types of environments are considered in this model. In the first type, licensing is not permitted. This extreme case is used to understand the strategic interactions between different firm-level decisions in an environment where there are institutional or technological impediments to licensing.

The other environment is one in which licensing is allowed. The licensing process is given. Each firm is a buyer with probability $\frac{1}{2}$ and a seller with probability $\frac{1}{2}$. The buyer firm offers to buy all the other firm's un-imitated ideas by proposing a payment that makes the seller indifferent between selling and not-selling. P_1^L is the payment offered by firm 1 when firm 1 is the buyer and P_2^L is the payment offered by firm 2 when firm 2 is the buyer. N_1^L denotes the number of ideas obtained by firm 1 (when firm 1 is the buyer) after successful licensing. The licensing process works in the same way for firm 2.

I also consider an alternative licensing environment which mimics the cross-licensing arrangements observed in the semiconductor industry. In this type of licensing (in this paper) firms share all their inventions and both firms go to the product development stage with the same number of total ideas. Each firm makes a payment to the other firm based on the number of their in-house ideas and imitated ideas.

Firms start developing the new product at the end of the second stage.

Stage 3 (Product Development Stage): At the beginning of this stage firms have either successfully completed the development of a new product or they have fail. The probability of successful product development by firm 1 is given by

$f_1 = f(N_1, (1-\phi_2)N_2)$, where N_1 is the number of new ideas invented by firm 1 and $(1-\phi_2)N_2$ is the number of firm 2's ideas imitated by firm 1. Firm 2 faces a similar problem. Once the new product is developed, profits are realized. The cost of production is zero. If only one firm is able to develop the product then that firm gets the monopoly revenue of $\pi_M > 0$.[5] If both firms develop the product simultaneously, they engage in Bertrand competition, which implies that each of them charges a price of zero for the product and earns zero profit. I assume that when the firms are indifferent between producing and not producing they choose to produce the amount dictated by consumer demand. If none of the firms are successful in developing the product then the revenue for each of them is zero.

3.1 Equilibrium

An equilibrium is defined here.

In stage 1, firms choose their R&D and patenting strategies before inventions are actually realized. Firms at this stage, therefore, maximize expected profit by taking into account that there will be a licensing stage after the inventions are realized.

If there is no licensing, firm 1's expected revenue conditional on the number of ideas of each firm is given by

$$\pi_1^{nl} = \pi_M f(N_1, (1-\phi_2)N_2)[1 - f(N_2, (1-\phi_1)N_1)], \tag{1}$$

where *nl* stands for the no-licensing case. The expected revenue structure for firm 2 is similar.

[5]The monopoly profit is obtained in the usual way from the consumer demand function $P = D(Q)$ and the cost of production of the firm. The cost of production is zero in this case.

If there is licensing and firm 1 is the buyer (which happens with probability $\frac{1}{2}$) then firm 1 offers to buy N_1^L ideas by making a payment P_1^L. P_1^L is a function of (N_1, N_2, N_1^L), and is chosen in accordance with the given licensing environment. An offer (N_1^L, P_1^L) is always accepted by the seller because of the structure of the offer. However, the buyer firm will make an offer only if $\pi_{1B} \geq 0$. For firm 1 that means the unconditional probability of success without licensing must be less than $\frac{1}{2}$. For a highly complex industry this condition will be satisfied. Hence for this paper the industry in question is assumed to be complex enough so that this condition is always satisfied. After licensing, firm 1 now has access to $(N_1 + (1-\phi_2)N_2 + N_1^l)$ ideas, while firm 2 has access to the same ideas as before, which is $(N_2 + (1-\phi_1)N_1)$. The probability that firm 1 succeeds in developing the final product and firm 2 fails is $f(N_1, (1-\phi_2)N_2, N_1^l)[1 - f(N_2, (1-\phi_1)N_1)]$. Firm 1's expected revenue as a buyer, conditional on the number of ideas of each firm, is

$$\pi_{1B} = \pi_M f(N_1, (1-\phi_2)N_2, N_1^L)[1 - f(N_2, (1-\phi_1)N_1)] - P_1^L(N_1, N_2, N_1^L) \quad (2)$$

such that

$$N_1^l \geq 0.$$

When firm 1 is the seller (which happens with probability $\frac{1}{2}$), firm 1 is offered (N_2^l, P_2^l)— a payment of P_2^l for licensing N_2^l ideas to firm 2. Here $N_2^l \geq 0$ and P_2^l is a function of (N_1, N_2, N_2^l). The offer is such that firm 1 always accepts. Therefore after licensing firm 1 has access to the same ideas as before $(N_1, (1-\phi_2)N_2)$, while firm 2 has now has access to $(N_2, (1-\phi_1)N_1, N_2^l)$ ideas. Hence, firm 1's expected revenue as a seller conditional on the number of ideas of each firm is

$$\pi_{1S} = \pi_M f(N_1, (1-\phi_2)N_2)[1 - f(N_2, (1-\phi_1)N_1, N_2^l)] + P_2^L(N_1, N_2, N_2^l) \quad (3)$$

such that

$$N_2^l \geq 0.$$

The equilibrium for this model is defined as follows :

DEFINITION : An *industry equilibrium* is a collection of R&D strategies $\{R_1^*, R_2^*\}$ and IP strategies $\{\phi_1^*, \phi_2^*\}$ which are obtained as follows:
Firm 1 chooses $R_1(R_2, \phi_2)$ and $\phi_1(R_2, \phi_2)$ by maximizing expected profit. The expectation is over the number of ideas for each firm conditional on R&D and patent protection. Firm 2 solves an identical problem and obtains $R_2(R_1, \phi_1)$ and $\phi_2(R_1, \phi_1)$. R_1^*, R_2^*, ϕ_1^* and ϕ_2^* are the Nash equilibrium values of R&D and patent protection.

I only look at the symmetric Nash equilibrium of the model to keep the analysis simple, i.e., $R_1^* = R_2^* = R^*$ and $\phi_1^* = \phi_2^* = \phi^*$. In a symmetric Nash equilibrium, the equilibrium solution can be expressed in terms of (R^*, ϕ^*).

3.2 The Optimal Solution

The optimal choice problem is formulated in the following way : the planner chooses R_1, R_2 and ϕ_1, ϕ_2 in stage 2 to maximize the expected sum of consumer surplus (CS) and producer surplus (PS). The planner intervenes at the R&D and patenting stages, but does not intervene in the product market. This assumption is maintained because the focus of this model is to understand the effects of the patent system on innovation and patenting only, and not on the market structure.

In the production stage there is no intervention. If only one firm innovates then the innovating firm gets the monopoly profit, π_M, the rival firm gets zero and the consumer surplus is small. If both firms innovate, then both firms engage in Bertrand competition, each get a revenue of zero and the consumer surplus is the maximum. If none of the firms innovate then both producer and consumer surplus are zero.

Therefore, the **optimal solution** is given by the set $\{R_1^{SP}, R_2^{SP}, \phi_1^{SP}, \phi_2^{SP}\}$ such that in stage 2, the planner chooses R_1, R_2 and ϕ_1, ϕ_2 that maximize the expected total surplus.

To make both the industry equilibrium and the planner's problem more tractable some specific functional forms are introduced.

3.3 Functional Assumptions

The following functional assumptions are made :

$f_i = f(N_i, N_i^{'}) = f(N_i + N_i^{'}) = 1 - (1-\lambda)^{(N_i + N_i^{'})} \quad \forall i, N_i, N_i^{'}, \quad 0 < \lambda < 1.$

$N_i^{'}$ takes the value of $(1-\phi_j)N_j, \ \forall i \neq j$ in the absence of licensing and takes the value $((1-\phi_j)N_j + N_i^l)$ in presence of licensing.

This particular functional assumption for $f(.)$ is useful because it allows the probability of success for a firm to be a function of the sum of ideas that are accessible to the firm. This implies that ideas are non-rival in terms of the role they play in the product development process. In the presence of this kind of non-rivalry it is expected that a stronger patent system will be useful as an instrument for exclusion and, hence, will raise the individual firm's incentive to invest in R&D. What this model demonstrates is that even when ideas are non-rival at the product development stage, under certain circumstances, the strengthening of the patent system may have less of an impact on an individual firm's incentive to invest in R&D.

The parameter λ can be interpreted as follows : λ is the probability of success for a firm which has access to only one new invention. λ is a constant for each industry. I call $(1-\lambda)$ the *complexity parameter* of the industry — for a firm that has access to only one idea in an industry parameterized by λ the chance of success is given by λ. In other words, firms in a *complex* product industry have a smaller value of λ than those in a *simple* product industry.[6] For firms to succeed in complex product industries it is, therefore, more important for them to have access to as many ideas as possible (compared to firms in simple-product industries).

The following functional form is chosen for $g(.)$:

$$g_i = g(R_i) = \begin{cases} 0 & \text{for } 0 \leq R_i < \frac{1}{\beta} \\ \ln(\beta R_i) & \text{for } R_i \geq \frac{1}{\beta}, \end{cases}$$

where $\beta > 0$ is a constant and $i = 1, 2$.

The following functional form is chosen for $C(.)$: $C(\phi) = \frac{1}{S}(\phi^2 + \phi)$, where S is a strictly positive parameter that represents the strength of the patent regime. A higher value of S implies a stronger patent regime in the sense that firms can obtain the same IP protection (ϕ) at a lower cost. The value of S is determined by the existing patent policy environment and is given in this model.

The model is solved for three different cases : 1) the 3-stage problem without licensing, 2) the planner's problem, where the planner makes R&D and patenting decisions, and 3) the 3-stage problem with licensing.

The no-licensing case is used as a benchmark. This case approximates an industry environment where licensing is not common due to either the history of development of the industry, or technological factors. The planner's problem gives the optimal level of R&D and patenting. The case with licensing is introduced to study complex product industries, like semiconductors, where cross-licensing and other multilateral arrangements to share technical knowledge are very com-

[6]The semiconductor industry is an example of a *complex* product industry, where new products typically embody many new ideas.

mon. Cross-licensing, in particular, is very common among semiconductor firms and has become more important in the recent years. Rival firms competing for the same market often share their technical know-how or simply give broad right-of-use over a bunch of patented and non-patented ideas.[7] After talking to the IP managers of some of the firms in complex industries, Cohen, Nelson and Walsh (2000) have suggested that the recent surge in patenting (after 1982) may be the result of strategic consideration by firms who want to have a better bargaining position in their cross-licensing arrangements. The reason for studying the licensing case is to understand how, in the presence of ex-post licensing, firms in complex product industries choose their R&D and patenting strategies and how the strategies change when the patent regime becomes more strong.

4 Equilibrium With No Licensing

In this case there are only two stages — the stage where the firms choose R&D expenditures and patenting strategies, and the product development stage. There is no licensing stage. To solve this model I start from the last stage with firm 1's problem. Firm 2 has identical problems at each stage.

In the last stage the profits are realized. Firms do not take any decisions at this stage.

In the first stage firm 1 chooses its R&D expenditure R_1 and patenting strategy ϕ_1. Since there is no licensing in this case firm 1 has access to only those ideas which it develops in-house, N_1, and to those it imitate $(1 - \phi_2)N_2$. At this stage,

[7] For example, competitors like Intel and AMD engage in cross-licensing arrangements regularly.

however, both N_1 and N_2 are random variables. The distribution of N_1 depends on the R&D expenditure R_1, while the distribution of N_2 depends on firm 2's R&D expenditure, R_2. The number of imitated ideas firm 1 can get depends on firm 2's patenting strategy, ϕ_2. Firm 1 similarly affects the number of ideas firm 2 can imitate by choosing patenting strategy, ϕ_1.

Firm 1 chooses R_1 and ϕ_1 by maximizing expected profit.

$$\max_{R_1,\phi_1} E\{\pi_1^{nl}\} - C(\phi_1) - R_1, \tag{4}$$

where π_1^{nl} stands for firm 1's revenue in the no-licensing (nl) case.

Since the revenue is a function of N_1 and N_2, which are random variables at this stage, firm 1 calculates expected revenue. A firm gets positive revenue only when it successfully develops the new product and the other firm fails to develop the new product. The probability that firm 1 succeeds with $(N_1 + (1 - \phi_2)N_2)$ ideas and firm 2 fails with $(N_2 + (1 - \phi_1)N_1)$ ideas is $f(N_1 + (1 - \phi_2)N_2)[1 - f(N_2 + (1 - \phi_1)N_1)]$. Therefore, expected revenue of firm 1 is

$$\pi_M E\{\pi_1^{nl}|R_1, R_2, \phi_1, \phi_2]\}, \tag{5}$$

where π_1^{nl} is given by (1). The above expectation is over the number of ideas for each firm conditional on R&D and patent protection.

Given the assumptions about the functional form of $f(.)$ and distribution of N_1, N_2, the expected revenue becomes:

$$\begin{aligned} & \pi_M E\{[1 - (1 - \lambda)^{N_1 + (1-\phi_2)N_2}](1 - \lambda)^{N_2 + (1-\phi_1)N_1}|R_1, R_2, \phi_1, \phi_2\} \qquad (6) \\ = \quad & \pi_M E\{(1 - \lambda)^{N_2 + (1-\phi_1)N_1} - (1 - \lambda)^{(2-\phi_1)N_1 + (2-\phi_2)N_2}|R_1, R_2, \phi_1, \phi_2\}. \end{aligned}$$

Now given the distributional assumptions,

$$
\begin{aligned}
E\{(1-\lambda)^{N_2+(1-\phi_1)N_1}\} &= E\{(1-\lambda)^{N_2}\}E\{(1-\lambda)^{(1-\phi_1)N_1}\} \quad (7)\\
&= [\sum_{N_2=0}^{\infty}\frac{(1-\lambda)^{N_2}e^{-g_2}g_2^{N_2}}{N_2!}][\sum_{N_1=0}^{\infty}\frac{(1-\lambda)^{(1-\phi_1)N_1}e^{-g_1}g_1^{N_1}}{N_1!}]\\
&= e^{-g_2\lambda}e^{-g_1 a(\phi_1)},
\end{aligned}
$$

where $a(\phi_1) = 1-(1-\lambda)^{(1-\phi_1)}$.

Similarly it can be shown that

$$E\{(1-\lambda)^{(2-\phi_1)N_1+(2-\phi_2)N_2}\} = e^{-g_1 b(\phi_1)}e^{-g_2 b(\phi_2)}, \quad (8)$$

where $b(\phi) = 1-(1-\lambda)^{(2-\phi)}$.

Therefore, firm 1's maximization problem is

$$
\begin{aligned}
\max_{R_1,\phi_1} \quad & \pi_M\{e^{-g_2\lambda}e^{-g_1 a(\phi_1)} - e^{-g_1 b(\phi_1)}e^{-g_2 b(\phi_2)}\} - C(\phi_1) - R_1 \quad (9)\\
\text{s.t.} \quad & 0 \le \phi_1 \le 1,\\
\& \quad & R_1 \ge 1/\beta.
\end{aligned}
$$

The Lagrangian for the above problem is:

$$
\begin{aligned}
L &= \pi_M\{e^{-g_2\lambda}e^{-g_1 a(\phi_1)} - e^{-g_1 b(\phi_1)}e^{-g_2 b(\phi_2)}\} - C(\phi_1) - R_1 \quad (10)\\
&\quad +\mu_1\phi_1 + \mu_2(1-\phi_1) + \theta_1(R_1 - \frac{1}{\beta}),
\end{aligned}
$$

where $\mu_1 \ge 0$, $\mu_2 \ge 0$, $\theta_1 \ge 0$ are Lagrange multipliers.

It can be shown that a unique global maxima exists only for small values of λ. Therefore for this analysis I choose a relatively more complex industry such that the global maxima exists for the above problem.

The first order conditions of the above problem are:

$$
\begin{aligned}
\frac{\partial L}{\partial \phi_1}: \quad & \pi_M g_1[-e^{-(g_2\lambda+g_1 a(\phi_1))}\frac{\partial a}{\partial \phi_1} + e^{-(g_1 b(\phi_1)+g_2 b(\phi_2))}\frac{\partial b}{\partial \phi_1}] \quad (11)\\
- \quad & \frac{\partial C}{\partial \phi_1} + \mu_1 - \mu_2 = 0
\end{aligned}
$$

$$\frac{\partial L}{\partial R_1}: \qquad \pi_M \frac{\partial g_1}{\partial R_1}\{-e^{-(g_2\lambda+g_1 a(\phi_1))}a(\phi_1)+e^{-(g_1 b(\phi_1)+g_2 b(\phi_2))}b(\phi_1)\} \qquad (12)$$
$$- \quad 1+\theta_1=0,$$

where $\mu_1 \geq 0$, $\mu_2 \geq 0$, $\theta_1 \geq 0$.

Firm 1 solves these two equations to obtain $\phi_1(\phi_2, R_2)$ and $R_1(\phi_2, R_2)$. Firm 2 solves a similar problem to obtain $\phi_2(\phi_1, R_1)$ and $R_2(\phi_1, R_1)$. In Nash equilibrium

$\phi_1^* = \phi_1(\phi_2^*, R_2^*), \quad \phi_2^* = \phi_2(\phi_1^*, R_1^*)$

$R_1^* = R_1(\phi_2^*, R_2^*), \quad R_2^* = R_2(\phi_1^*, R_1^*)$.

The functional forms of $a(.)$ and $b(.)$ are

$$a(\phi) = 1-(1-\lambda)^{(1-\phi)}, \qquad (13)$$

$$b(\phi) = 1-(1-\lambda)^{(2-\phi)}. \qquad (14)$$

For a symmetric Nash equilibrium the first order conditions, (11) and (12), give

$$\pi_M g(1-\lambda)^{(1-\phi)}\ln(1-\lambda)[-e^{-g(\lambda+1-(1-\lambda)^{(1-\phi)})} \qquad (15)$$
$$+(1-\lambda)e^{-2g(1-(1-\lambda)^{(2-\phi)})}] - \frac{\partial C}{\partial \phi} + \mu_1 - \mu_2 = 0,$$

$$\pi_M \frac{1}{R}[-e^{-g(\lambda+1-(1-\lambda)^{(1-\phi)})}(1-(1-\lambda)^{(1-\phi)}) \qquad (16)$$
$$+e^{-2g(1-(1-\lambda)^{(2-\phi)})}(1-(1-\lambda)^{(2-\phi)})] - 1 = 0,$$

where $\mu_1 \geq 0, \mu_2 \geq 0$.

The first order condition identifies three distinct regions of S, separated by two cut-off points $\underline{S}^{nl}$ and $\bar{S}^{nl}$, with $\bar{S}^{nl} > \underline{S}^{nl}$. If the regime parameter $S < \underline{S}^{nl}$, then firm 1 will choose $\phi_1 = 0$. For $\underline{S}^{nl} < S < \bar{S}^{nl}$, firm 1 will choose ϕ_1, such that

$0 < \phi_1 < 1$. For $S > \bar{S}^{nl}$, firm 1 will choose $\phi_1 = 1$. Similar results are obtained for firm 2. These results are summarized in the following proposition:

Proposition 1. For each industry λ and for each firm i, there are two cut-off points $\underline{S}^{nl}$ and $\bar{S}^{nl}$, with $\bar{S} > \underline{S}$ such that

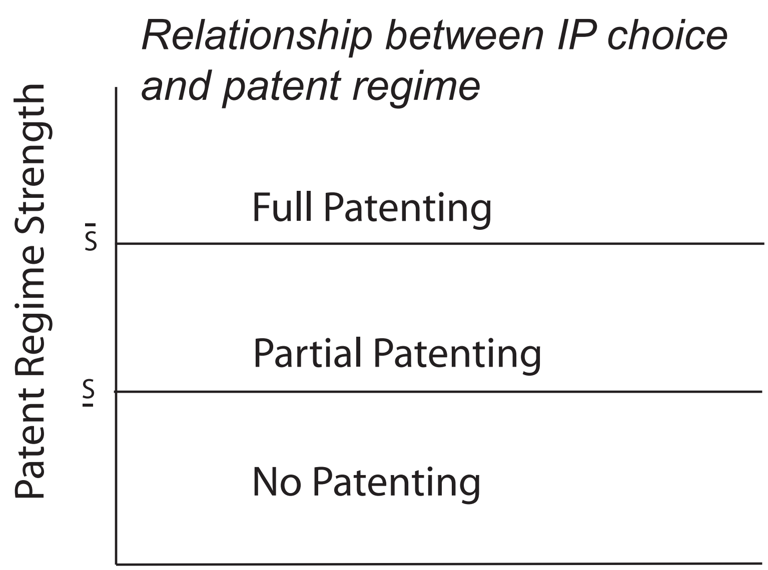
i) for all $S < \underline{S}^{nl}$, firm i will choose $\phi_i = 0$,

ii) for all $\underline{S}^{nl} < S < \bar{S}^{nl}$, firm i will choose $0 < \phi_i < 1$, and

iii) for all $S > \bar{S}^{nl}$, firm i will choose $\phi_i = 1$.

Proof: See Appendix A.

The above results are intuitive. If the IP regime is very weak (S below a certain cut-off value) then for each firm in a particular industry the marginal cost of enforcing IP will exceed the marginal gain from enforcing IP rights. For those low values of S, firms will choose no IP protection at all. Similarly, in a very strong IP regime the marginal gains exceed marginal cost and hence firms choose the highest IP protection possible.



The above problem is now solved for three different patent regimes — strong patent regime ($S > \bar{S}^{nl}$), weak patent regime ($S < \underline{S}^{nl}$) and moderate patent regime ($\underline{S}^{nl} < S < \bar{S}^{nl}$).

4.1 Case 1 : Strong patent regime

When firms choose $\phi_1 = \phi_2 = 1$ from (16) it follows that

$$\pi_M \lambda \frac{1}{R} e^{-2g\lambda} = 1, \tag{17}$$

or, (given the functional assumption on $g(.)$)

$$\pi_M \lambda \frac{1}{R} (\beta R)^{-\lambda} (\beta R)^{-\lambda} = 1. \tag{18}$$

Therefore, the R&D expenditure of each firm under a very strong patent regime in the no-licensing case is given by:

$$R^{nl}_{\{\phi=1\}} = [\lambda \pi_M \beta^{-2\lambda}]^{\frac{1}{1+2\lambda}} \tag{19}$$

4.2 Case 2 : Weak patent protection

When firms choose $\phi_1 = \phi_2 = 0$ equation (16) gives:

$$\pi_M \lambda \frac{1}{R} e^{-2g\lambda} [-1 + (2-\lambda) e^{-2g\lambda(1-\lambda)}] = 1. \tag{20}$$

The expression $[-1+(2-\lambda)e^{-2g\lambda(1-\lambda)}]$ is denoted by δ. Since $(2-\lambda)e^{-2g\lambda(1-\lambda)} < 2$, it must be that $\delta < 1$.

Therefore, using the functional assumption on $g(.)$,

$$\pi_M \lambda (\beta R)^{-2\lambda} \delta = 1, \tag{21}$$

or,

$$R^{nl}_{\{\phi=0\}} = [\lambda \pi_M \beta^{-2\lambda} \delta]^{\frac{1}{1+2\lambda}} \text{ where } \quad \delta < 1. \tag{22}$$

Therefore $R^{nl}_{\{\phi=0\}} < R^{nl}_{\{\phi=1\}}$.

4.3 Case 3 : Moderate patent regime

I discuss the most general case here. For this part the following approximations for a and b are made: for small λ

$$a = 1 - (1-\lambda)^{(1-\phi)} \approx 1 - (1 - \lambda(1-\phi)) = \lambda(1-\phi), \tag{23}$$

$$b = 1 - (1-\lambda)^{(2-\phi)} \approx 1 - (1 - \lambda(2-\phi)) = \lambda(2-\phi). \tag{24}$$

Using the above approximations the first order conditions, (15) and (16), give:

$$\pi_M g \lambda e^{-(2-\phi)g\lambda}[1 - e^{-(2-\phi)g\lambda}] = \frac{\partial C}{\partial \phi} = \frac{2\phi+1}{S}, \tag{25}$$

$$\pi_M \lambda \frac{1}{R} e^{-(2-\phi)g\lambda}[-(1-\phi) + (2-\phi)e^{-(2-\phi)g\lambda}] = 1. \tag{26}$$

From these first order conditions the following results are obtained:

Proposition 2. For small values of λ,

i) $\frac{\partial R_1^*}{\partial \phi_1^*} > 0$,

ii) $\frac{\partial \phi_1^*}{\partial S} > 0$.

Proof: See Appendix A

From proposition 2 we get that the patenting strategy of a firm and its R&D strategy are strategic complements. When firms choose strong IP protection they also have less access to imitated ideas and, hence, must also choose higher R&D to obtain a greater number of in-house ideas. This is because the probability of success for a firms depends on the total number of ideas available to it. Strong patent protection restricts access to ideas invented by other firms and, hence, increases the expected returns on in-house R&D in the margin. On the other hand the relation between the strength of the patent regime and firms choice of IP protection is positive. Firms choose higher levels of patent protection when the external patent regime strengthens. These results can be combined to get the following proposition:

Corollary 1: A stronger patent regime in an industry with no licensing will lead to more patent protection and more R&D.

This result is in tune with what economists have presented as the main reason for having the institution of patent. When ideas are non-rival and can be imitated at a small cost, the resulting positive externality provides incentives for individual firms to invest less in R&D to produce new ideas. The institution of patents takes care of this by awarding ownership rights. Patents, therefore, take care of the free rider problem and boost investment in R&D.

In addition the following results are also obtained:

Proposition 3.

i) $\frac{\partial S}{\partial \lambda} < 0$ for all λ, i.e., firms in more complex industries will choose $\phi = 0$ for a larger range of S-values, and

ii) $\frac{\partial \bar{S}}{\partial \lambda} < 0$ for all λ, i.e., firms in more complex industries will choose $\phi = 1$ for a smaller range of S-values.

Proof: See Appendix A.

The intuition is that firms in smaller λ industries produce complex products that require using a relatively large number of ideas. These firms would, therefore, choose the weakest IP protection ($\phi = 0$) for a wider range of IP regimes as compared to firms in higher λ industries.

In the case with no licensing firms in complex industries are unable to capture all of the positive externality of the knowledge pool created at the industry level. Next I consider the social planner's problem.

5 The Planner's Problem

The consumers are represented by a linear demand : $P(Q) = a - bQ$, with $a, b > 0$. The planner maximizes the sum of the producer surplus and the consumer surplus.

Three distinct cases can arise : i) only one firm successfully develops the product, ii) both firms successfully develop the product, and iii) none of the firms are successful. The total surplus will be different in each of these three cases.

Case i) If one firm successfully develops the final product, there will be a monopoly. The probability that only firm will successfully develop the final product is $\{f(N_1, (1 - \phi_2)N_2)[1 - f(N_2, (1 - \phi_1)N_1)] + f(N_2, (1 - \phi_1)N_1)[1 - f(N_1, (1 - \phi_2)N_2)]\}$.

Assuming zero production cost, the monopoly profit is obtained by

$$\max_{Q}(a - bQ)Q. \tag{27}$$

Using the functional assumptions, the above maximization gives a producer surplus of $\frac{a^2}{4b}$ and a consumer surplus of $\frac{a^2}{8b}$. The producer surplus is denoted by π_M, which also denotes the monopoly revenue following the notation introduced in the previous section. In this case the producer surplus is π_M, the consumer surplus is $\frac{1}{2}\pi_M$ and the total surplus is $\frac{3}{2}\pi_M$.

Therefore the total expected revenue in this case is

$$\begin{aligned} \pi_i^{SP} &= \frac{3}{2}\pi_M\{f(N_1 + (1-\phi_2)N_2)[1 - f(N_2 + (1-\phi_1)N_1)] \\ &+ f(N_2 + (1-\phi_1)N_1)[1 - f(N_1 + (1-\phi_2)N_2)]\}. \end{aligned} \tag{28}$$

Case ii) When both firms successfully develop the final product, the firms will engage in Bertrand type competition, that will result in the minimum producer surplus and the maximum consumer surplus. The probability that both firms will successfully develop the final product is given by $f(N_1+(1-\phi_2)N_2)f(N_2+(1-\phi_1)N_1)$.

In this case the producer surplus is 0, the consumer surplus is $2\pi_M$, and the total surplus is $2\pi_M$.

Therefore the total expected revenue in this case is

$$\pi_{ii}^{SP} = 2\pi_M f(N_1 + (1-\phi_2)N_2)f(N_2 + (1-\phi_1)N_1). \tag{29}$$

Case iii) If both firms fail to develop the final product then both the producer surplus and the consumer surplus, and hence the total surplus, will be equal zero. This will happen with probability $[1-f(N_1+(1-\phi_2)N_2)][1-f(N_2+(1-\phi_1)N_1)]$.

Therefore the planner chooses :

$$\{R_1^{SP}, R_2^{SP}, \phi_1^{SP}, \phi_2^{SP}\} = \text{argmax } E[\pi_i^{SP} + \pi_{ii}^{SP}] - C(\phi_1) - C(\phi_2) - R_1 - R_2 \quad (30)$$

s.t. $0 \leq \phi_1 \leq 1, \quad 0 \leq \phi_2 \leq 1, \quad R_1 \geq 1/\beta, \quad R_2 \geq 1/\beta$,

where π_i^{SP} and π_{ii}^{SP} are given by (28) and (29).

The maximization problem can be rewritten as

$$\begin{aligned} &\max_{R_1,R_2,\phi_1,\phi_2} \pi_M E\{\frac{3}{2}[1-(1-\lambda)^{N_1+(1-\phi_2)N_2}](1-\lambda)^{N_2+(1-\phi_1)N_1} \quad (31)\\ &+\frac{3}{2}[1-(1-\lambda)^{N_2+(1-\phi_1)N_1}](1-\lambda)^{N_1+(1-\phi_2)N_2}\\ &+2[1-(1-\lambda)^{N_1+(1-\phi_2)N_2}][1-(1-\lambda)^{N_2+(1-\phi_1)N_1}]\}\\ &-C(\phi_1)-C(\phi_2)-R_1-R_2. \end{aligned}$$

Now the expected revenue part can be simplified as follows:

$$\begin{aligned} &\pi_M[-\frac{1}{2}(1-\lambda)^{N_2+(1-\phi_1)N_1} - \frac{1}{2}(1-\lambda)^{N_1+(1-\phi_2)N_2} - (1-\lambda)^{(2-\phi_1)N_1+(2-\phi_2)N_2}\\ = \; &\pi_M[-\frac{1}{2}e^{-(g_2\lambda+g_1a(\phi_1))} - \frac{1}{2}e^{-(g_1\lambda+g_2a(\phi_2))} - e^{-(g_1b(\phi_1)+g_2b(\phi_2))}]. \end{aligned}$$

The Lagrangian for this problem is

$$\begin{aligned} L \; = \; &\pi_M[-\frac{1}{2}e^{-(\lambda g_2+g_1a(\phi_1))} - \frac{1}{2}e^{-(\lambda g_1+g_2a(\phi_2))} - e^{-(g_1b(\phi_1)+g_2b(\phi_2))}]\\ - \; &C(\phi_1) - C(\phi_1) - R_1 - R_2\\ + \; &\eta_1\phi_1 + \eta_2(1-\phi_1) + \delta_1\phi_2 + \delta_2(1-\phi_2) + \gamma_1(R_1 - \frac{1}{\beta}) + \gamma_2(R_2 - \frac{1}{\beta}), \end{aligned}$$

where $\eta_1 \geq 0, \eta_2 \geq 0, \delta_1 \geq 0, \delta_2 \geq 0, \gamma_1 \geq 0, \gamma_2 \geq 0$ are Lagrange multipliers.

The planner's problem gives the following proposition.

Proposition 4. The planner will choose

i) $\phi_1^{SP} = \phi_2^{SP} = 0$, for all values of S and

ii) $R^{SP} > R^{nl}_{\phi=1}$.

Proof: See Appendix A.

The above results are intuitive. The planner gets the maximum social surplus when there is competition, that is, when both firms successfully develop the product. Therefore, the planner would favor the minimum IP protection that would allow firms to tap into each others' knowledge pool and increase the probability of successful product development. R&D investment is an expenditure for the firm, but in deciding the level of optimal R&D the planner takes into account both the producer surplus and the consumer surplus. The consumer surplus is an increasing function of R&D expenditure. The consumers derive surplus from R&D, but they do not incur the R&D expenditure. Therefore, the planner would want more R&D than what each firm would choose on their own because the planner problem has this additional consumer-surplus component.[8]

Next I discuss the case where ex-post licensing of inventions is allowed.

6 Case 2 : Equilibrium with Licensing

In this case there are three stages. In stage one firms decide IP and R&D strategies. Inventions are realized and imitated at the end of the first stage. In stage two one firm offers to be the licensee of the un-imitated ideas owned by other firm, provided that licensing is profitable for the licensee.[9] Licensing gives a firm the

[8]Note that the planner's decision is independent of the patent regime. Hence a change in the patent regime would not change anything in the planner's problem.

[9]Licensees are referred to as the buyers and licensors as the sellers in this paper.

opportunity to access the others firm's inventions. In stage three, firms develop products and the profits are realized. The firms do not make any decisions at this stage.

A very specific form of licensing is considered here. After the invention and imitation stage the firms go to the licensing stage. Each firm is a buyer of ideas with probability $\frac{1}{2}$ and a seller of ideas with probability $\frac{1}{2}$. The buyer firm makes a *take-it-or-leave-it* offer to the other firm to acquire all of the other firm's un-imitated ideas. The offer includes a payment that makes the seller firm indifferent between selling and not selling. This licensing structure has been chosen for its simplicity. I do not address the issue of firms' choice of licensing mechanism since that is not the focus of this paper.

It is important to note that the specific structure of the licensing arrangement is such that the seller always accept an offer and the buyer is better off licensing as long as the payoff to the buyer under the no-licensing regime is positive. It has been already discussed that in the no-licensing case the expected payoff will be positive for firms in industries with small values of λ. Hence the buyers and sellers in sufficiently complex industries will choose to engage in licensing when they enter the licensing stage, regardless of how many total ideas they have acquired (even for $N_1 = 0$ or $N_2 = 0$).

The model is solved backwards, starting from stage three. In stage three firms undertake production and profits are realized. At the beginning of stage two, either firm 1 or 2 makes a *take-it-or-leave-it* offer to the other firm. The licensing mechanism is such that an offer, once made, is always accepted. A new product is developed (or not) at the end of this period. In stage one firms choose their patenting and licensing strategies.

I start with firm 1's problem. In stage three, profits are realized and the firms do not make any decisions. In stage two, given N_1, N_2, firm 1 is the buyer of ideas with probability $\frac{1}{2}$ and seller with probability $\frac{1}{2}$. As a buyer, firm 1 offers P_1^L to acquire all of firm 2's ideas. Similarly as a seller firm 1 receives an offer of P_2^L as a payment for selling all its ideas to firm 2.

When firm 1 is the buyer (which happens with probability $\frac{1}{2}$) the expected revenue conditional on the number of ideas of each firm is

$$\begin{aligned} \pi_{1B} &= \pi_M f(N_1 + N_2)[1 - f(N_2 + (1-\phi_1)N_1)] - P_1^L \\ &\text{s.t.} \quad P_1^L \leq \pi_M f(N_2 + (1-\phi_1)N_1)[f(N_1+N_2) - f(N_1 + (1-\phi_2)N_2)]. \end{aligned} \tag{32}$$

When firm 1 is the seller (which happens with probability $\frac{1}{2}$) the expected revenue conditional on the number of ideas of each firm is

$$\begin{aligned} \pi_{1S} &\geq \pi_M f(N_1 + (1-\phi_2)N_2)[1 - f(N_2 + N_1)] + P_2^L \\ &\text{s.t.} \quad P_2^L = \pi_M f(N_1 + (1-\phi_2)N_2)(f(N_2+N_1) - f(N_2 + (1-\phi_1)N_1)). \end{aligned} \tag{33}$$

Hence in stage one, given that firms decide to engage in licensing in stage two, firm 1 solves for :

$$\begin{aligned} \{R_1^*, \phi_1^*\} &= \text{argmax} \frac{1}{2} E[\pi_{1B} + \pi_{1S} | R_1, \phi_1, R_2^*, \phi_2^*] - C(\phi_1) - R_1 \\ &\text{s.t.} \quad P_1^L = f(N_2 + (1-\phi_1)N_1)[f(N_1+N_2) - f(N_1 + (1-\phi_2)N_2)] \\ &\& \quad P_2^L = f(N_1 + (1-\phi_2)N_2)(f(N_2+N_1) - f(N_2 + (1-\phi_1)N_1)) \\ &\& \quad 0 \leq \phi_1 \leq 1, \quad R_1 \geq 1/\beta. \end{aligned} \tag{34}$$

The expectation in (34) is over the number of ideas of each firm conditional on R&D and patenting. The expected revenue part of the above equation gives

$$\begin{aligned}\frac{1}{2}E[\pi_{1B}+\pi_{1S}] &= \frac{\pi_M}{2}E\{2(1-\lambda)^{(1-\phi_1)N_1+N_2}-2(1-\lambda)^{(2-\phi_1)N_1+2N_2} \\ &\quad -(1-\lambda)^{N_1+(1-\phi_2)N_2}+(1-\lambda)^{N_1+N_2}\} \\ &= \frac{\pi_M}{2}\{2e^{-g_2\lambda}e^{-g_1a(\phi_1)}-2e^{-g_1b(\phi_1)}e^{-2g_2\lambda} \\ &\quad -e^{-g_1\lambda}e^{-g_2a(\phi_2)}+e^{-g_1\lambda}e^{-g_2\lambda}\},\end{aligned}$$

where $a(\phi)=1-(1-\lambda)^{(1-\phi)}$ and $b(\phi)=1-(1-\lambda)^{(2-\phi)}$.

The Lagrangian of the above equation is given by:

$$\begin{aligned}L &= \frac{\pi_M}{2}\{2e^{-g_2\lambda}e^{-g_1a(\phi_1)}-2e^{-g_1b(\phi_1)}e^{-2g_2\lambda}-e^{-g_1\lambda}e^{-g_2a(\phi_2)}+e^{-(g_1+g_2)\lambda}\} \\ &\quad -C(\phi_1)-R_1+\mu_3\phi_1+\mu_4(1-\phi_1)+\theta_2(R_1-\frac{1}{\beta}),\end{aligned} \tag{35}$$

where $\mu_3\geq 0, \mu_4\geq 0, \theta_2\geq 0$ are Lagrange multipliers.

The first order conditions are:

$$\begin{aligned}\phi_1: &\quad \frac{\pi_M}{2}2g_1\{-e^{-(g_2\lambda+g_1a(\phi_1))}\frac{\partial a}{\partial\phi_1}+e^{-(g_1b(\phi_1)+2g_2\lambda)}\frac{\partial b}{\partial\phi_1}\} \\ - &\quad \frac{\partial C}{\partial\phi_1}+\mu_3-\mu_4=0,\end{aligned} \tag{36}$$

$$\begin{aligned}R_1: &\quad \frac{\pi_M}{2}\frac{\partial g_1}{\partial R_1}\{-2e^{-(g_2\lambda+g_1a(\phi_1))}a(\phi_1)+2e^{-(g_1b(\phi_1)+2g_2\lambda)}b(\phi_1) \\ + &\quad \lambda e^{-(\lambda g_1+g_2a(\phi_2))}-\lambda e^{(g_1+g_2)\lambda}\}-1+\theta_2=0.\end{aligned} \tag{37}$$

For the symmetric equilibrium equations (36) and (37) give

$$\pi_M g\lambda e^{-(2-\phi)g\lambda}[1-e^{-2g\lambda}]-\frac{\partial C}{\partial\phi}+\mu_3-\mu_4=0, \tag{38}$$

$$\frac{\pi_M}{2}\frac{1}{R}\lambda[(2\phi-1)e^{-(2-\phi)g\lambda}+2(2-\phi)e^{-(4-\phi)g\lambda}-e^{-2g\lambda}]-1\theta_2=0. \quad (39)$$

The first order conditions again identify three distinct regions of the S-line, separated by two cut-off points $\underline{S}^l$ and $\bar{S}^l$. For all $S > \bar{S}^l$, firms choose the maximum patent protection ($\phi_1 = \phi_2 = 1$). For all $S < \bar{S}^l$, firms choose the minimum patent protection ($\phi_1 = \phi_2 = 0$).

I again start by solving the model for three different patent regimes — a strong patent regime, a weak patent regime and a moderate patent regime.

6.1 Case 1 : Strong patent regime

When the patent regime is very strong, firms choose $\phi_1 = \phi_2 = 1$.

From the first order condition:

$$\frac{\pi_M}{2}\lambda\frac{1}{R}[e^{-g\lambda}+2e^{-3g\lambda}-e^{-2g\lambda}]=1. \quad (40)$$

For small values of λ it can be shown

$$R^l_{\{\phi=1\}} < R^{nl}_{\{\phi=1\}}, \quad (41)$$

that is, the R&D chosen by firms at the maximum level of IP protection is lower in the presence of licensing as compared to the no-licensing case.

6.2 Case 2 : Weak patent regime

When the patent regime is very weak, firms choose $\phi_1 = \phi_2 = 0$.

From the first order condition:

$$\pi_M\lambda\frac{\partial g}{\partial R}e^{-2g\lambda}[2e^{-2g\lambda}-1]=1. \quad (42)$$

Comparing this with the no-licensing case I find that

$$R^{l}_{\{\phi=0\}} = R^{nl}_{\{\phi=0\}}, \tag{43}$$

that is, the R&D chosen by firms at the minimum level of IP protection in the presence of bargaining is equal to that of the no-bargaining case.

Thus, so far it has been shown that when firms choose the lowest IP protection ($\phi = 0$), their R&D expenditure choice remains the same in both the no-licensing and licensing equilibrium. This is not surprising, because the licensing stage is trivial when there are no ideas (in expected terms) to license and, hence, the two cases give identical results.

The most general case is considered next.

6.3 Case 3 : Moderate patent regime

For interior solution, the first order conditions for a symmetric equilibrium are

$$\pi_M g\lambda e^{-(2-\phi)g\lambda)}[1 - e^{-2g\lambda}] - \frac{\partial C}{\partial \phi_1} = 0, \tag{44}$$

$$\frac{\pi_M}{2}\frac{\lambda}{R}[(2\phi - 1)e^{-(2-\phi)g\lambda} + 2(2-\phi)e^{-(4-\phi)g\lambda} - e^{-2g\lambda}] = 1. \tag{45}$$

Analyzing the above equations the following results are obtained for firms in complex industries:

Proposition 5.

i) For small values of λ, $\frac{\partial R}{\partial \phi}$ will be positive, but $\frac{\partial R}{\partial \phi}|_l < \frac{\partial R}{\partial \phi}|_{nl}$, where nl stands for the no-licensing case and l stands for the licensing case, and

ii) $\frac{\partial \phi}{\partial S}$ will be positive.

Proof: See Appendix A.

This proposition states that although a strengthening of the patent regime will always lead to higher patenting and higher R&D, the strategic complementarity between R&D decision and patenting decision of a firm is weakened in the presence of licensing. Even if both types of licensing environments generate the same increase in patenting in response to a stronger patent regime change, the R&D increment will be smaller in an industry where licensing is widespread.

Firms in complex industries rely heavily on licensing to tap into other firm's ideas. As the surveys mentioned before suggest, licensing has become a very important for firms in the semiconductor industry after the 1982-change in the U.S. patent regime. The relatively small increase in the aggregate R&D in this industry then may be due to the relatively weak complementarity between a firm's R&D decisions and patenting decision. In the presence of licensing, a firm in a complex industry has access to an additional mechanism for obtaining other firm's inventions and will, therefore, have less incentive to invest in R&D to generate in-house inventions. A stronger patent regime will increase patenting, but the corresponding increase in R&D will be smaller in presence of licensing than what would have been in the absence of licensing.

6.4 A Numerical Experiment

As suggested by Grindley and Teece (1997), the U.S. patent regime change had an impact on the licensing environment of the semiconductor industry. The change in the patent regime seems to have increased licensing and cross-licensing activities in the semiconductor industry. In light of the above observation, I consider the pre-1982 licensing environment as one that approximates to the no-licensing case described in this paper and the post-1982 licensing environment approximates to the with-licensing case. The patent regime has changed more things than just the regime parameter; it has also changed the licensing environment.

In this section, I report a numerical exercise to understand the impact on firm-level patenting and research variables due a joint change in the patent regime and the licensing environment. The parameter values chosen are reported below.
The complexity parameter is chosen to be $\lambda = 0.005$ so as to represent complex industries, like semiconductor, electronics, etc.[10] The profit (π_M) is chosen to be 10 million.[11] To obtain values for the β parameter, the R&D is taken to be a fifth of the total profit. The average number of patents in the semiconductor industry for 2002 is about 55. This gives a parameter value of β in the order of 10^{17}. For this exercise, $\beta_1 = \beta_2 = 3 \times 10^{17}$ are chosen. The parameter S relates to the cost of patenting. A parameter value of $S = 2 \times 10^{-6}$ implies that the cost of complete patent protection ($\phi = 1$) for a firm runs in the order of a million. $S = 2 \times 10^{-6}$ is considered a weak patent regime. A 50% increase in the regime parameter constitutes a stronger patent regime. The results obtained are summarized in the

[10]The MPEG4 Visual Patent Portfolio contains approximately 180 patents (http://www.mpegla.com/m4v/index.cfm). This is not a final product for the consumers but a compression technology that allows developers of web streaming and videophone to develop their final products. A firm in an industry with $\lambda = 0.005$ that acquires all of these 180 patents will have 60% chance of developing a final product.

[11]The general conclusion of this experiment remains same for higher or lower values of profit. However, keeping all other parameters the same, a very small profit drives the patenting parameter to zero, while a very large profit makes the patenting parameter equal to 1.

following table:

	Weak Patent Regime	Stronger Patent Regime
No Licensing	ϕ (patenting) = 0.15 g (research) = 54.15	
With Licensing		ϕ (patenting) = 0.69 g (research) = 54.08

The above exercise shows that a change towards a stronger patent regime change that also alters the licensing environment will have a large positive impact on firm-level patenting decision, but may have only a small (and negative) impact on the research decision. This tallies well with the data from the semiconductor industry where the post-1982 large increase in patenting has not been matched by a similar large increase in research. It is possible that the *patent paradox* is a product of the changed licensing environment that has followed the patent regime change. A strong licensing environment coupled with a strong intellectual property regime has enhanced the importance of patenting for firms by making the size of the patent portfolio an important determinant of the terms of the licensing, but has not changed the incentives to conduct R&D significantly.

7 Conclusion

The strengthening of the U.S. patent regime after 1982 was followed by a large increase in the number of patents and by an unchanging R&D expenditure trend, particularly in the complex product industries. Whether these observations can be explained by studying the effects of a stronger patent regime on an industry environment where bilateral licensing of technologies is common, is that main focus of this paper. The model presented here shows that for complex product industries, where bilateral licensing is common, a stronger patent-regime change will have a smaller impact on the firm-level R&D decision compared to that in other industries where licensing is less important.

The impact of the licensing environment on firm-level R&D decision is probably also a function of the size of the firm in terms of its stock of patents. Larger firms with an already large patent portfolio might enjoy a better bargaining position more frequently and, hence, their firm-level R&D decision might be less sensitive to a change in the patent regime. The patenting and R&D data for the semiconductor industry shows that the four largest firms (in the sample considered in Dey (2005)) in the semiconductor industry have increased both their patenting activity as well as their R&D activity substantially after the 1982-change in the U.S. patent regime.[12] The patent paradox does not seem to be holding for this group of firms, while it definitely holds for the medium and the small-sized firms. Since this model considers homogenous firms, the size-effect is not captured in the model. This remains a project for the future.

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Appendix A

Proof of Proposition 1

From equation (15) for $\phi_1 = \phi_2 = 0$ (i.e., $\mu_2 = 0$),

$$\pi_M g(1-\lambda)ln(1-\lambda)[-e^{-2g\lambda} + (1-\lambda)e^{-2g\lambda(2-\lambda)}] - \frac{\partial C}{\partial \phi_1} + \mu_1 = 0, \quad (46)$$

where $\mu_1 > 0$.

Therefore,

$$\frac{\partial C}{\partial \phi_1}|_{\phi_1=0} = \frac{1}{S} > \pi_M g^0(1-\lambda)\ln(1-\lambda)[-e^{-2g^0\lambda} + (1-\lambda)e^{-2g^0\lambda(2-\lambda)}], \quad (47)$$

or,

$$S < [\pi_M g^0(1-\lambda)\ln(1-\lambda)(-e^{-2g^0\lambda} + (1-\lambda)e^{-2g^0\lambda(2-\lambda)})]^{-1}, \quad (48)$$

where $g^0 = g|_{\phi=0}$. The right hand side of the above expression gives $\underline{S}^{nl}$.

Similarly, for $S > \bar{S}^{nl}$ given by $\bar{S}^{nl} = 3[\pi_M g^1 \ln(1-\lambda)(-e^{-g^1\lambda} + (1-\lambda)e^{-2g^1\lambda})]^{-1}$, where $g^1 = g|_{\phi=1}$, firms will choose $\phi_1 = \phi_2 = 1$.■

Expressions for g^1 and g^0 are solved in the next 4.1 and 4.2 respectively.

Proof of Proposition 2

i) Equation (26) gives

$$\pi_M \lambda e^{-(2-\phi)g\lambda}\{(2-\phi)e^{-(2-\phi)g\lambda} - 1 + \phi\}\frac{\partial g}{\partial R} = 1. \quad (49)$$

Differentiating w.r.t. ϕ, the following result is obtained for small values of λ:

$$-\frac{\partial R}{\partial \phi}\frac{1}{R}[1 + (2-\phi)\lambda + (2-\phi)^2\lambda] + g\lambda[-(1-\phi) + 2(2-\phi)] = 0 \quad (50)$$

or,

$$\frac{\partial R}{\partial \phi} \mid_{nl} = \frac{(3-\phi)g\lambda R}{1+\lambda(2-\phi)(3-\phi)}. \tag{51}$$

Since $\phi \leq 1$ it must be that $\frac{\partial R}{\partial \phi} > 0$.

ii) Differentiating equation (25) w.r.t. S:

$$\pi_M \lambda e^{-(2-\phi)g\lambda} \frac{\partial g}{\partial \phi} \frac{\partial \phi}{\partial S} [(1-e^{-(2-\phi)g\lambda})(1-g\lambda(2-\phi)) + g\lambda e^{-(2-\phi)g\lambda}(2-\phi)]$$
$$+ \frac{\partial \phi}{\partial S} [\pi_M (g\lambda)^2 e^{-(2-\phi)g\lambda}(1-2e^{-(2-\phi)g\lambda}) - \frac{2}{S}] = -\frac{(2\phi+1)}{S^2}.$$

For small values of λ, the following result is obtained from the above equation:

$$\frac{\partial \phi}{\partial S} [\pi_M g\lambda((2-\phi)\lambda \frac{\partial g}{\partial \phi} - g\lambda) - \frac{2}{S}] = -\frac{2\phi+1}{S^2} \tag{52}$$

As $\lambda \to 0$, the negative term on the l.h.s. dominates the positive term. Since the function is continuous, for small values of λ the bracketed term on the l.h.s. is negative. Since the r.h.s. is also negative, it must be that $\frac{\partial \phi}{\partial S} > 0$.■

Proof of Proposition 3

i) $\underline{S} = [\pi_M g^0 \lambda (1-e^{-2g^0\lambda}) e^{-2g^0\lambda}]^{-1}$.

Now, $R^0 = [\lambda \pi_M \beta^{-2\lambda} \delta]^{\frac{1}{1+2\lambda}}$, where $\delta < 1$. Therefore

$$\begin{aligned} g^0 &= \ln(\beta R^0) \\ &= \ln[\lambda \pi_M \beta \delta]^{\frac{1}{1+2\lambda}} \\ &= \frac{1}{1+2\lambda} ln(\lambda \pi_M \beta \delta) \end{aligned} \tag{53}$$

Therefore,

$$\begin{aligned}
\frac{\partial g^0}{\partial \lambda} &= -\frac{ln(\lambda \pi_M \beta \delta)}{(1+2\lambda)^2} + \frac{1}{(1+2\lambda)\lambda} \\
&= \frac{1}{(1+2\lambda)}[\frac{1}{\lambda} - g^0]
\end{aligned} \tag{54}$$

Differentiating $\underline{S}$ w.r.t. λ gives

$$\begin{aligned}
\frac{\partial \underline{S}}{\partial \lambda} &= -\pi_M e^{-2g^0\lambda}(\frac{\partial g^0}{\partial \lambda}\lambda + g^0)[1 - e^{-2g^0\lambda} + 2g^0\lambda(2e^{-2g^0\lambda} - 1)] \\
&= -\pi_M e^{-2g^0\lambda}(\frac{1}{1+2\lambda}(\frac{1}{\lambda} - g^0)\lambda + g^0)[1 - e^{-2g^0\lambda} + 2g^0\lambda(2e^{-2g^0\lambda} - 1)] \\
&= -\pi_M e^{-2g^0\lambda}(\frac{1}{1+2\lambda}(\frac{1}{\lambda} - g^0)\lambda + g^0)[1 - e^{-2g^0\lambda} + 2g^0\lambda(2e^{-2g^0\lambda} - 1)]
\end{aligned} \tag{55}$$

Now, $(\frac{1}{1+2\lambda}(\frac{1}{\lambda} - g^0)\lambda + g^0) = 1 + g^0 + g^0\lambda > 0$. For $\lambda \to 0$, it can be shown that $[1 - e^{-2g^0\lambda} + 2g^0\lambda(2e^{-2g^0\lambda} - 1)] > 0$. Hence for small values of λ, $\frac{\partial \underline{S}}{\partial \lambda} < 0$.■

Proof of Proposition 4

i) Note that the above objective function is a monotonically decreasing function of ϕ_1, ϕ_2. Hence the planner will choose $\phi_1^{SP} = \phi_2^{SP} = 0$.

ii) The first order conditions for R_1 and R_2 are given by

$$\begin{aligned}
R_1 : \quad & \pi_M \frac{\partial g_1}{\partial R_1}[\frac{1}{2}e^{-(\lambda g_2 + g_1 a(\phi_1))}a(\phi_1) + \frac{1}{2}e^{-(\lambda g_1 + g_2 a(\phi_2))}\lambda] \\
& + e^{-(g_1 b(\phi_1) + g_2 b(\phi_2))}b(\phi_1)] - 1 = 0,
\end{aligned} \tag{56}$$

$$\begin{aligned}
R_2 : \quad & \pi_M \frac{\partial g_2}{\partial R_2}[\frac{1}{2}e^{-(\lambda g_2 + g_1 a(\phi_1))}\lambda + \frac{1}{2}e^{-(\lambda g_1 + g_2 a(\phi_2))}a(\phi_2)] \\
& + e^{-(g_1 b(\phi_1) + g_2 b(\phi_2))}b(\phi_2)] - 1 = 0.
\end{aligned} \tag{57}$$

Therefore, in the symmetric equilibrium

$$\pi_M \lambda \frac{1}{R} e^{-2g\lambda}[1+(2-\lambda)e^{-2g\lambda(1-\lambda)}] = 1. \tag{58}$$

I define $\psi = [1+(2-\lambda)e^{-2g\lambda(1-\lambda)}]$. Now $\psi > 1$ for all values of g.

Therefore, using the functional assumptions, the R&D expenditure chosen by the planner for each firm is given by:

$$R^{SP} = [\lambda \pi_M \beta^{-2\lambda} \psi]^{\frac{1}{1+2\lambda}}, \text{where } \ \psi > 1. \tag{59}$$

Therefore, $R^{SP} > R^{nl}_{\phi=1}$.■

Proof of Proposition 5

i) Differentiating (44) with respect to ϕ

$$\begin{aligned} &\frac{\pi_M \lambda}{2R^2}\frac{\partial R}{\partial \phi}[-(2\phi-1)e^{-(2-\phi)g\lambda} - 2(2-\phi)e^{-(4-\phi)g\lambda} + e^{-2g\lambda} \\ &-(2\phi-1)(2-\phi)\lambda e^{-(2-\phi)g\lambda} - 2(2-\phi)(4-\phi)\lambda e^{-(4-\phi)g\lambda} + 2\lambda e^{-2g\lambda}] \\ &= \frac{\pi_M \lambda}{2R}[-2e^{-(2-\phi)g\lambda} - (2\phi-1)g\lambda e^{-(2-\phi)g\lambda} + 2e^{-(4-\phi)g\lambda} - 2(2-\phi)g\lambda e^{-(2-\phi)g\lambda}] \end{aligned} \tag{60}$$

For small λ the exponential terms can be approximated by 1, which gives

$$\frac{\partial R}{\partial \phi}\mid_l = \frac{3g\lambda R}{2+7\lambda(2-\phi)-2\lambda}. \tag{61}$$

Comparing this with the no-licensing case, it can be shown that $\frac{\partial R}{\partial \phi}|_l < \frac{\partial R}{\partial \phi}|_{nl}$.

ii) Differentiating equation (45) w.r.t. S gives

$$\begin{aligned} &\lambda^2 \pi_M \frac{\partial g}{\partial \phi}\frac{\partial \phi}{\partial S} e^{-(2-\phi)g\lambda}[1-e^{-2g\lambda} - (2-\phi)g(1-e^{-2g\lambda}) + 2ge^{-2g\lambda}] \\ &+\pi_M g^2 \lambda^2 e^{-(2-\phi)g\lambda}(1-e^{-2g\lambda})\frac{\partial \phi}{\partial S} = \frac{2}{S}\frac{\partial \phi}{\partial S} - \frac{2\phi+1}{S^2} \end{aligned} \tag{62}$$

For small λ,

$$\frac{\partial \phi}{\partial S}\{2\pi_M g\lambda^2\frac{\partial g}{\partial \phi} - \frac{2}{S}\} = -\frac{2\phi + 1}{S^2}. \tag{63}$$

For small λ the term inside the parenthesis on the left-hand side will be negative. Therefore, it must be that $\frac{\partial \phi}{\partial S} > 0$.■